A note on terminology: Zeros and roots are the same thing. If they are real, as opposed to complex, they are also x-intercepts of your graph.

Write the equation in its equivalent exponential or logarithmic form:

To convert between an exponential expression and a logarithmic expression, it is often helpful to use the "first-last-middle" rule to perform the conversion. If necessasy, set the expression equal to x before applying the rule.

Note: the "first-last-middle" rule requires that the logarithmic or exponential portion of the expression be on the left-hand side of the equation.

Converting from Logarithmic Form to Exponential Form

$$\log_b a = x$$

$$b^x = a$$

using "first-last-middle"

Examples:

1) Solve for x: $\log_4 64 = x$.

   First is "4", **last** is "x" and **middle** is "64." So, $4^x = 64$.

   Then, $4^1 = 4;\ 4^2 = 16;\ 4^3 = 64$ ✓

   So, we have: $\boldsymbol{x = 3}$

2) Solve for x: $\ln e = x$ (remember *ln* is shorthand for $\log_e$)

   Using first-last-middle,

   $\log_e e = x$ converts to: $e^x = e$

   So, we have: $\boldsymbol{x = 1}$

Converting from Exponential Form to Logarithmic Form

$$b^x = a$$

$$\log_b a = x$$

using "first-last-middle"

Examples:

1) Convert the expression, $2^5 = 32$ to logarithmic form.

   First is "2", **last** is "32" and **middle** is "5".

   So, we have: $\boldsymbol{\log_2 32 = 5}$

2) Convert the expression, $7^3 = 343$ to logarithmic form.

   Using first-last-middle,

   $7^3 = 343$ converts to: $\log_7 343 = 3$

   So, we have: $\boldsymbol{\log_7 343 = 3}$

Calculator allowed only on problems marked with an asterisk *.

For #1 – 4: Convert the equation to its equivalent exponential or logarithmic form:

1. $\log_b 64 = 2$

   In the log expression, $\log_b 64 = 2$, **first** is "b", **last** is "2" and **middle** is "64." We put these in an exponential expression, from left to right, to get: $\boldsymbol{b^2 = 64}$.

2. $\log_6 216 = x$

In the log expression, $\log_6 216 = x$, **first** is "6", **last** is "x" and **middle** is "216." We put these in an exponential expression, from left to right, to get: $\mathbf{6^x = 216}$.

3. $2^3 = x$

First-last-middle works this way too.

In the exponential expression, $2^3 = x$, **first** is "2", **last** is "x" and **middle** is "3." We put these in a logarithmic expression, from left to right, to get: $\mathbf{\log_2 x = 3}$.

4. $2^{-2} = \frac{1}{4}$

In the exponential expression, $2^{-2} = \frac{1}{4}$, **first** is "2", **last** is "$\frac{1}{4}$" and **middle** is "-2." We put these in a logarithmic expression, from left to right, to get: $\mathbf{\log_2 \left(\frac{1}{4}\right) = -2}$.

For #5 – 12: Evaluate each expression without the use of a calculator.

5. $\log 10$

In the log expression, $\log_{10} 10 = x$, **first** is "10", **last** is "x" and **middle** is "10." We put these in an exponential expression, from left to right, to get: $10^x = 10$, then solve:

$\log_{10} 10 = x$ converts to: $10^x = 10$ ⟶ $\mathbf{x = 1}$

6. $\log_3 \sqrt{3}$

In the log expression, $\log_3 \sqrt{3} = x$, **first** is "3", **last** is "x" and **middle** is "$\sqrt{3}$." We put these in an exponential expression, from left to right, to get: $3^x = \sqrt{3}$, then solve:

$\log_3 \sqrt{3} = x$ converts to: $3^x = \sqrt{3}$ ⟶ $\mathbf{x = \frac{1}{2}}$

Note: $\sqrt{3} = 3^{1/2}$

7. $\log_6 1$

In the log expression, $\log_6 1 = x$, **first** is "6", **last** is "x" and **middle** is "1." We put these in an exponential expression, from left to right, to get: $6^x = 1$, then solve:

$\log_6 1 = x$ converts to: $6^x = 1$ $\longrightarrow$ $\boldsymbol{x = 0}$

Problems 8 to 9: Exponentiation and taking logarithms are inverse operations, so when they both exist, *with the same base*, they cancel each other out.

8. $6^{\log_6 15} - \ln e^2$

$$6^{(\log_6 15)} - \ln e^2 = 15 - 2 = \mathbf{13}$$

Note: By using parentheses, we can make it easier to see what is going on in the problem.

9. $\log_7 49^9$

Tip: If the argument of a log can be expressed in terms of the base of the log, do that before proceeding.

$$\log_7(49^9) = \log_7((7^2)^9) = \log_7(7^{18}) = \mathbf{18}$$

10. $\log_3\left(\frac{1}{27}\right)$

$$\log_3\left(\frac{1}{27}\right) = \log_3(3^{-3}) = \mathbf{-3}$$

11. $\frac{1}{2}\log_2 64$

$$\frac{1}{2}\log_2(64) = \frac{1}{2}\log_2(2^6) = \frac{1}{2}(6) = \mathbf{3}$$

12. $\ln e + 8^{\log_2 5} - \log 1000$

$$\ln e + 8^{\log_2 5} - \log 1000$$

$$= \log_e e + (2^3)^{\log_2 5} - \log_{10} 10^3$$

$$= \log_e e + (2^{\log_2 5})^3 - \log_{10} 10^3$$

$$= 1 + (5)^3 - 3$$

$$= 1 + 125 - 3$$

$$= \mathbf{123}$$

For #13 – 19: Expand or condense the following expressions:

13. $\log_2 xy$

$$\log_2 xy = \mathbf{\log_2 x + \log_2 y}$$

14. $\log_5\left(\frac{a^2}{x}\right)$

$$\log_5\left(\frac{a^2}{x}\right) = \log_5 a^2 - \log_5 x = \mathbf{2\log_5 a - \log_5 x}$$

15. $\ln(\sqrt[3]{y} \cdot z^4)$

$$\ln(\sqrt[3]{y}\, z^4) = \ln(y^{1/3}) + \ln(z^4) = \mathbf{\frac{1}{3}\ln y + 4\ln z}$$

16. $\log_4\left(\frac{x-6}{x^5}\right)$

$$\log_4\left(\frac{x-6}{x^5}\right) = \log_4(x-6) - \log_4(x^5) = \mathbf{\log_4(x-6) - 5\log_4 x}$$

17. $\log_4(x-8) - \log_4(x-4)$

$$\log_4(x-8) - \log_4(x-4) = \mathbf{\log_4\left(\frac{x-8}{x-4}\right)}$$

18. $3\log_6 x + 5\log_6(x-6)$

$$3\log_6 x + 5\log_6(x-6) = \log_6 x^3 + \log_6(x-6)^5 = \mathbf{\log_6[x^3(x-6)^5]}$$

19. $4\ln 2 - \ln 8$

$$4\ln 2 - \ln 8 = \ln 2^4 - \ln 2^3 = \ln\frac{2^4}{2^3} = \mathbf{\ln 2}$$

Alternatively,

$$4\ln 2 - \ln 8 = 4\ln 2 - \ln 2^3 = 4\ln 2 - 3\ln 2 = \mathbf{\ln 2}$$

For #20 – 29: Solve the following equations. No calculator unless marked with an asterisk *(#29… round to 3 decimal places.) If no calculator, give exact answer.

20. $4^{(1+2x)} = 64$

Starting equation:	$4^{(1+2x)} = 64$
Take the "$\log_4$" of both sides:	$\log_4\left(4^{(1+2x)}\right) = \log_4(64)$
Simplify:	$1 + 2x = 3$
Subtract 1:	$2x = 2$
Divide by 2:	$\boldsymbol{x = 1}$

21. $e^{(x+8)} = \frac{1}{e^4}$

Starting equation:	$e^{(x+8)} = \frac{1}{e^4}$
Convert exponent on the right:	$e^{(x+8)} = e^{-4}$
Take the "ln" of both sides:	$x + 8 = -4$
Subtract 8:	$\boldsymbol{x = -12}$

22. $5^{(x+7)} = 3$

Starting equation:	$5^{(x+7)} = 3$
Take the "ln" of both sides:	$\ln\left(5^{(x+7)}\right) = \ln(3)$
Simplify:	$(x+7)\ln 5 = \ln 3$
Divide by $\ln 5$:	$x + 7 = \frac{\ln 3}{\ln 5}$
Subtract 7:	$\boldsymbol{x = \frac{\ln 3}{\ln 5} - 7}$

Note: I tend to use ln in solving problems like this. However, you can use logs with any base. Other useful bases for this problem might be log, $\log_3$ or $\log_5$.

23. $e^{2x} - 11e^{x} + 24 = 0$

Treat this as a quadratic equation in e^x.

Starting equation:	$e^{2x} - 11e^{x} + 24 = 0$	
Make a quadratic equation:	$(e^{x})^2 - 11e^{x} + 24 = 0$	
Factor the equation:	$(e^{x} - 3)(e^{x} - 8) = 0$	
Set each term equal to zero:	$(e^{x} - 3) = 0$	$(e^{x} - 8) = 0$
Isolate the e^x term:	$e^{x} = 3$	$e^{x} = 8$
Take the $\ln$ of both sides:	$\ln e^{x} = \ln 3$	$\ln e^{x} = \ln 8$
Simplify:	$x = \ln 3$	$x = \ln 8$
Collect solutions:	$\boldsymbol{x = \ln 3 , \ln 8}$	

24. $\log_3(x-1) = -1$

Starting equation:	$\log_3(x-1) = -1$
Take 3 to the power of both sides:	$3^{\log_3(x-1)} = 3^{-1}$
Simplify:	$x - 1 = \frac{1}{3}$
Add 1:	$\boldsymbol{x = \frac{4}{3}}$

25. $4 + 8\ln x = 8$

Starting equation:	$4 + 8\ln x = 8$
Subtract 4:	$8\ln x = 4$
Divide by 8:	$\ln x = \frac{1}{2}$
Take e to the power of both sides:	$e^{\ln x} = e^{1/2}$
Simplify:	$\boldsymbol{x = e^{1/2} = \sqrt{e}}$

26. $\log_6 x + \log_6(x-35) = 2$

Starting equation:	$\log_6 x + \log_6(x-35) = 2$
Combine log terms:	$\log_6[x(x-35)] = 2$
Take 6 to the power of both sides:	$6^{\log_6[x(x-35)]} = 6^2$
Simplify:	$x(x-35) = 36$
Multiply terms:	$x^2 - 35x = 36$
Subtract 36:	$x^2 - 35x - 36 = 0$
Factor:	$(x-36)(x+1) = 0$
Determine solutions for x:	$x = \{36, -1\}$
Test the solutions of x:	$x = 36$: $\log_6 36 + \log_6(36-35) = 2$ ✓
	$x = -1$: $\log_6(-1) + \log_6(-1-35) = 2$ ✗

Note: To test the solutions you derive, use the original equation or a simplified form of the original equation.

These terms are both Invalid because negative numbers are not in the domain of the log function.

Final solution: $x = 36$

Note: Generally, a log problem that involve a quadratic equation needs to have its solutions checked. One or both solutions may be extraneous, i.e., they solve the quadratic, but they do not solve the original problem.

27. $\log_6(5x-5) = \log_6(3x+7)$

Starting equation:	$\log_6(5x-5) = \log_6(3x+7)$
Take 6 to the power of both sides:	$6^{\log_6(5x-5)} = 6^{\log_6(3x+7)}$
Simplify:	$5x - 5 = 3x + 7$
Add 5:	$5x = 3x + 12$
Subtract $3x$:	$2x = 12$
Divide by 2:	$x = 6$
Test the solution of x:	$\log_6(5 \cdot 6 - 5) = \log_6(3 \cdot 6 + 7)$ ✓

Final solution: $x = 6$

28. $\log_8(x+5) + \log_8 x = \log_8 14$

Starting equation:	$\log_8(x+5) + \log_8 x = \log_8 14$ ← Use this equation to test your solutions below.
Combine log terms:	$\log_8[(x+5)x] = \log_8 14$
Take 8 to the power of both sides:	$8^{\log_8[(x+5)x]} = 8^{\log_8 14}$
Simplify:	$(x+5)x = 14$
Multiply terms:	$x^2 + 5x = 14$
Subtract 14:	$x^2 + 5x - 14 = 0$
Factor:	$(x-2)(x+7) = 0$
Determine solutions for x:	$x = \{2, -7\}$
Test the solutions of x:	$x = 2$: $\log_8(2+5) + \log_8 2 = \log_8 14$ ✓
	$x = -7$: $\log_8(-7+5) + \log_8(-7) = \log_8 14$ ✗
Final solution: $\boldsymbol{x = 2}$	These terms are both Invalid because negative numbers are not in the domain of the log function.

*29. $4^{(x+9)} = 5^{3x-7}$

Starting equation:	$4^{(x+9)} = 5^{3x-7}$
Take the $\ln$ of both sides:	$(x+9)\ln 4 = (3x-7)\ln 5$
Distribute:	$(\ln 4)x + 9\ln 4 = (3\ln 5)x - 7\ln 5$
Collect x-terms and constants:	$(\ln 4 - 3\ln 5)x = -7\ln 5 - 9\ln 4$
Divide by $(\ln 4 - 3\ln 5)$:	$x = \dfrac{-7\ln 5 - 9\ln 4}{\ln 4 - 3\ln 5}$
Final solution:	$\boldsymbol{x \sim 6.898}$

For #30 – 33: Graph each function. Include the asymptote and the anchor point. Also, describe the domain and range in interval notation.

Tip: start with the parent function, then reflect and translate as necessary. Check to see if points on the curve satisfy the equation if you are unsure about your answer.

For an exponential function of the form: $f(x) = b^{x-h} + k$, start with the parent function: $f(x) = b^x$, then shift the function h units right (shift left if h is negative) and k units up (shift down if k is negative). The horizontal asymptote is at $y = k$. One way to do all of this is to:

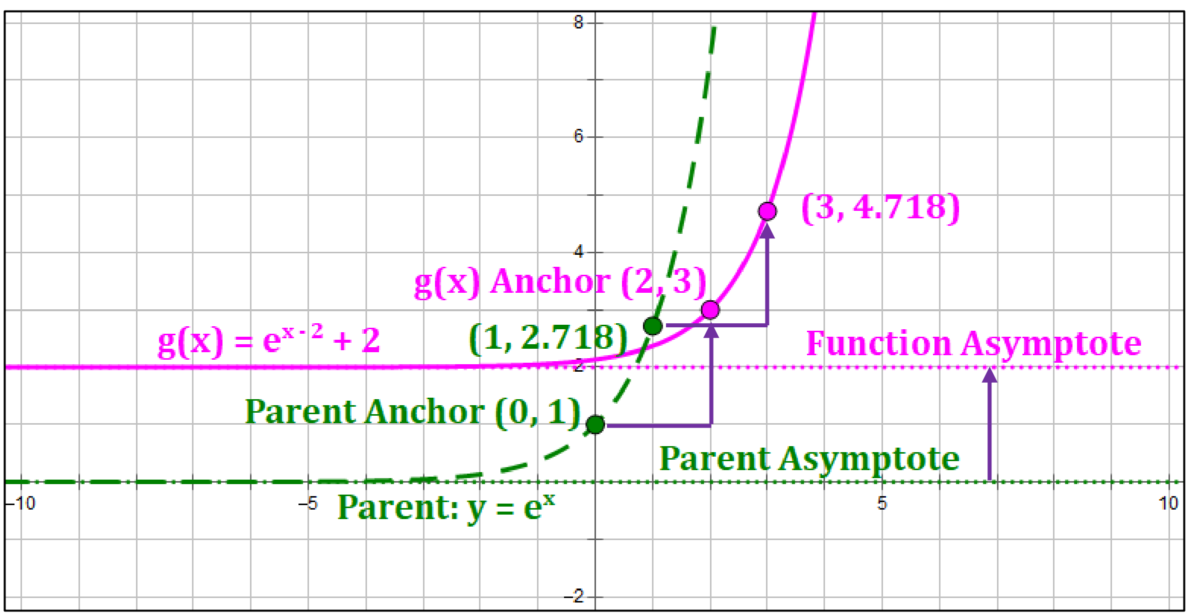
- Identify two points and any asymptotes on the parent function graph.
- Shift the two points and any asymptotes to set up the desired graph.
- Graph the desired function.
- Note: in the graphs that follow, shifts are shown with purple arrows. ⟶

30) $g(x) = e^{x-2} + 2$

The parent function is $y = e^x$. Estimate e as 2.718.

Shift the parent function 2 units right and 2 units up. Shift key points also, to help you draw $g(x)$.

Shift the horizontal asymptote of the parent, $y = 0$, 2 units up to become $y = 2$.



Domain: $(-\infty, \infty)$ always for a simple exponential function.

Range: $(2, \infty)$ always open and either beginning or ending with the asymptote value.

Anchor point: $(2, 3)$ translated point for parent anchor (0,1).

31) $y = -e^{-x} + 3$

Using the parent function approach on this problem would require two reflections and a translation. Yuk! It is much easier to draw with an asymptote and a few points.

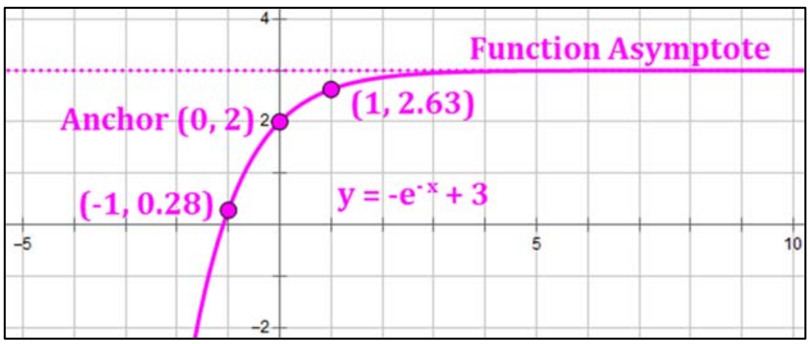
First draw in the horizontal asymptote @ $y = 3$.

Next, pick some strategic x-values, say $-1, 0, 1$ (centered on the value that makes the exponent 0)

Find the y-values for each selected x-value: Estimate e as 2.718.

- $x = -1$, $y = -e^{1} + 3 = 3 - e \sim 0.28$, Point is $(-1, 0.28)$
- $x = 0$, $y = -e^{0} + 3 = 3 - 1 = 2$, Point is $(0, 2)$ also, this is the anchor point
- $x = 1$, $y = -e^{-1} + 3 = 3 - \frac{1}{e} \sim 2.63$, Point is $(1, 2.63)$

Plot the asymptote and the 3 points, then run a smooth curve through them.



Domain: $(-\infty, \infty)$

Range: $(-\infty, 3)$

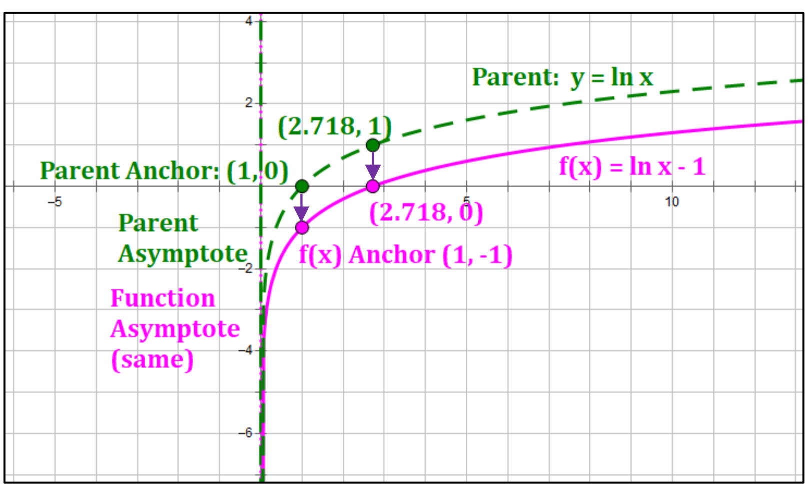
Anchor point: $(0, 2)$ Note, the anchor point is the one with an x-value that makes the exponent 0.

32) $f(x) = \ln x - 1$

The parent function is $f(x) = \ln x$. Estimate e as 2.718.

Shift 1 unit down.

Parent and $f(x)$ asymptotes are the same (vertical line $x = 0$, i.e., the y-axis).



Domain: $(0, \infty)$ always open and either beginning or ending with the vertical asymptote value.

Range: $(-\infty, \infty)$ always for a simple logarithmic function.

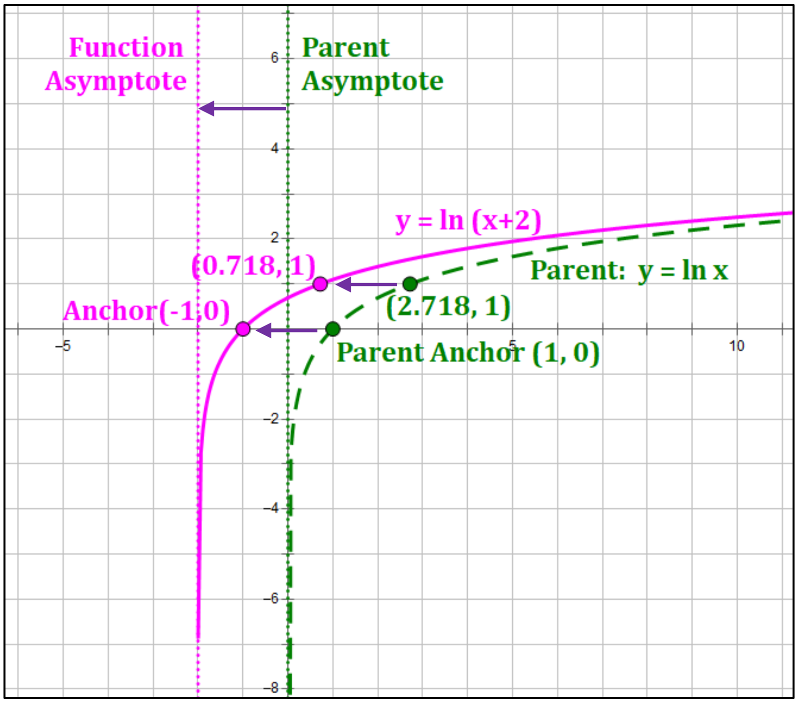
Anchor point: $(1, -1)$ translated point for parent anchor $(1, 0)$.

33) $y = \ln(x+2)$

The parent function is $y = \ln x$. Estimate e as 2.718.

Shift the parent function 2 units left. There is no vertical shift.

Shift the vertical asymptote of the parent, $x = 0$, 2 units left to become $x = -2$.



Domain: $(-2, \infty)$

Range: $(-\infty, \infty)$

Anchor point: $(-1, 0)$

*34) The brewing pot temperature of coffee is 180º F, and the surrounding room temperature is 76º F. After 5 minutes, the temperature of the coffee is 168º F. Use Newton's Law of Cooling: $T = C + (T_0 - C)e^{kt}$.

a) Write an equation to represent this situation.

There are two steps to problems like this:

1) Find the value of k based on the change in temperature over the first 5 minutes.
2) Write the equation using the value of k.

What are the variables?

$\boldsymbol{T}$ is the temperature of the object (coffee) at any point in time. $\boldsymbol{T}$ changes as $\boldsymbol{t}$ (time) changes.

$\boldsymbol{C}$ is the current temperature of the environment (e.g., the room).

$\boldsymbol{T_0}$ is the starting temperature of the object (coffee).

$\boldsymbol{k}$ is the rate of change in the temperature of the object (coffee) over time.

$\boldsymbol{t}$ is the time elapsed since the event (removal from the heat source) occurred. Initially, $\boldsymbol{t} = 0$.

We are given: $T_0 = 180$ $C = 76$ $t = 5$ $T = 168$

Starting equation: $T = C + (T_0 - C) \cdot e^{k \cdot t}$

Substitute known values: $168 = 76 + (180 - 76) \cdot e^{k \cdot 5}$

Simplify: $92 = (104) \cdot e^{5k}$

Divide by 104: $0.88462 = e^{5k}$

Take natural logs: $\ln(0.88462) = \ln(e^{5k})$

Simplify: $-0.12260 = 5k$

Divide by 5 to obtain k: $k = -0.02452$

Equation: $\boldsymbol{T = 76 + 104e^{-0.02452 \cdot t}}$

b) How long will it take for the coffee to reach a serving temperature of 155º F? Round your answer to one decimal place.

Starting equation: $155 = 76 + 104e^{-0.02452 \cdot t}$ (from previous problem)

Subtract 76: $79 = 104e^{-0.02452 \cdot t}$

Divide by 104: $0.75962 = e^{-0.02452 \cdot t}$

Take natural logs: $-0.27494 = -0.02452 \cdot t$

Divide by -0.02452 to obtain t: $\boldsymbol{t = 11.2}$ **minutes**

For #35 – 36: Use the compound interest formulas $A = P\left(1+\frac{r}{n}\right)^{nt}$ and $A = Pe^{rt}$ to solve:

35) Find the total value of an investment of \$900 at 12% compounded quarterly for 6 years.

We are given: $P = \$900$ $r = 0.12$ $n = 4$ (quarterly compounding)

$t = 6$ years $\cdot$ 4 quarters $= 24$ periods

$$A = P\left(1+\frac{r}{n}\right)^{nt} = 900\cdot\left(1+\frac{0.12}{4}\right)^{24} = 900\cdot(1.03)^{24} = \$1,829.51$$

Below is a disagram to help understand why we use $(1.03)^{24}$ for the interest factor.

12% per year, compounded quarterly, is 3% per quarter. Interest is compounded at the end of each quarter, of which there are $6\cdot 4 = 24$ quarters. The resulting factor is $(1+.03)^{24}$.

	Qtr 1	Qtr 2	Qtr 3	Qtr 4
Year 1	3%	3%	3%	3%
Year 2	3%	3%	3%	3%
Year 3	3%	3%	3%	3%
Year 4	3%	3%	3%	3%
Year 5	3%	3%	3%	3%
Year 6	3%	3%	3%	3%

6 years is equivalent to 24 quarters.

12% per year is equivalent to 3% per quarter.

36) Find the accumulated (total) value of an investment of \$4000 at 7.3% compounded continuously for 5 years.

Continuous compounding tells us to grab the shampoo! Use the "Pert" formula. Note: Pert is a brand of shampoo.

We are given: $P = \$4,000$ $r = 0.073$ continuous compounding

$t = 5$ years

$$A = Pe^{rt} = 4000\cdot\left(e^{5\cdot 0.073}\right) = 4000\cdot\left(e^{0.365}\right) = \$5,762.06$$

37. The half-life of Silicon-32 is 710 years. If 50 grams is present now, how much will be present in 200 years? (Round your answer to three decimal places.)

There are two steps to problems like this:

1) Find the value of k based on the half-life of 710 years.
2) Find how much would be left after 200 years.

What are the variables?

The formula for exponential decay is: $A = A_0 e^{kt}$, where:

- A is the amount of substance left at time t.
- A_0 is the starting amount of the substance.
- k is the annual rate of decay.
- t is the number of years.

Step 1: Determine the value of k (using the half-life of Silicon-32)

We are given: $t = 710$, $\frac{A}{A_0} = \frac{1}{2}$ (because we are given a "half-life")

Starting equation: $A = A_0 e^{kt}$

Divide by A_0: $\frac{A}{A_0} = e^{kt}$

Substitute in values: $\frac{1}{2} = e^{710\,k}$

Take natural logs: $\ln \frac{1}{2} = 710\,k$

Divide by 710: $k = \frac{\ln \frac{1}{2}}{710} = -0.00097626$

Step 2: Find how much is left after 200 years

We are given: are given: $t = 200$, $A_0 = 50$ grams $k = -0.00097626$

Starting equation: $A = A_0 e^{kt}$

Substitute in values: $A = 50 \cdot e^{(-0.00097626)\cdot 200} = \mathbf{41.131}$ **grams**

38. The logistic growth function $f(t) = \frac{87{,}000}{1+1449e^{-1.2t}}$ models the number of people who have become ill with a particular infection t weeks after its initial outbreak in a particular community.

a) How many people were ill after 9 weeks? Round your answer to the nearest whole number, if needed.

This is a simple substitution problem. Substitute 9 for t and calculate the solution.

$$f(9) = \frac{87{,}000}{1+1{,}449 \cdot e^{(-1.2)\cdot 9}} = \frac{87{,}000}{1+1{,}449 \cdot e^{-10.8}} = \mathbf{84{,}502\ people}$$

b) What is the max number of people that could get this infection? (In other works, what is the limiting value?

If we let t get larger and larger, the denominator of $f(t)$ gets closer and closer to 1. As the denominator approaches 1, $\boldsymbol{f(t)}$**, the population, approaches but never quite reaches** $\mathbf{87{,}000}$**.** **The limiting value is** $\mathbf{87{,}000}$**.**

39) The population of a particular country was 30 million in 1984; in 1989 it was 37 million. The exponential growth function $A = 30e^{kt}$ models the population of this country t years after 1984. Use the fact that 5 years after 1984 the population increased by 7 million to find k to three decimal places.

There is only one step to this problem:

1) Find the value of k based on the population change from 1984 to 1989.

What are the variables?

The formula for exponential decay is: $A = A_0 e^{kt}$, where:

- A is the population at time t.
- A_0 is the starting population.
- k is the annual rate of growth.
- t is the number of years.

Step 1: Determine the value of $\boldsymbol{k}$

We are given: $A = 37$, $A_0 = 30$, $t = 5$

Starting equation: $A = A_0 e^{kt}$

Substitute in values: $37 = 30e^{k \cdot 5}$

Divide by 30: $\frac{37}{30} = e^{5k}$

Take natural logs: $\ln\left(\frac{37}{30}\right) = 5k$

Divide by 5: $k = \frac{\ln\left(\frac{37}{30}\right)}{5} = \mathbf{0.0419441}$

Round to 3 decimal places: $\boldsymbol{k = 0.042}$

Note: We were aked to find the value of k to 3 decimal places, which we did. If you were asked to use k in further calculations, you should use as much precision (i.e., as many decimals) in the value of k as possible. This will result in more accurate solutions in the further calculations.

40) Carbon-14 has a half-life of 5600 years. A fossilized leaf contains 18% of its normal amount of carbon 14. How old is the fossil (to the nearest year)? Use an exponential decay model to solve this problem.

There are two steps to problems like this:

1) Find the value of k based on the half-life of 5600 years.
2) Find how old the fossil is when there is 18% left.

What are the variables?

The formula for exponential decay is: $A = A_0 e^{kt}$, where:

- A is the amount of substance left at time t.
- A_0 is the starting amount of the substance.
- k is the annual rate of decay.
- t is the number of years.

Step 1: Determine the value of k (using the half-life of Carbon-14)

We are given: $t = 5600$, $\frac{A}{A_0} = \frac{1}{2}$ (because we are given a "half-life")

Starting equation: $A = A_0 e^{kt}$

Divide by A_0: $\frac{A}{A_0} = e^{kt}$

Substitute in values: $\frac{1}{2} = e^{5600\,k}$

Take natural logs: $\ln\frac{1}{2} = 5600\,k$

Divide by 5600: $k = \frac{\ln\frac{1}{2}}{5600} = -0.000123776$

Note: If you round k differently, you will get a slightly different answer, e.g., 13,829 years. The answer given below provides the solution obtained by keeping all digits in the calculator throughout the entire calculation, rounding only in the final value, 13,854 years. interestingly, in science class, you would use 2 significant digits, and your answer would be "approximately 14,000 years."

Step 2: Find how old the fossil is when there is 18% left.

We are given: are given: $\frac{A}{A_0} = 18\%$ left,

$k = -0.000123776$

Starting equation: $A = A_0 e^{kt}$

Divide by A_0: $\frac{A}{A_0} = e^{kt}$

Substitute in values: $0.18 = e^{(-0.000123776)\cdot t}$

Take natural logs: $\ln(0.18) = -0.000123776 \cdot t$

Divide by (-0.000123776): $t = \frac{\ln(0.18)}{-0.000123776} = 13{,}854$ **years old**